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Abstract

Blundell, Pistaferri, and Preston (2008) report an estimate of household consumption insurance with respect to permanent income shocks of 36%. Their estimate is distorted by an error in their code and is not robust to the weighting scheme for GMM. We propose instead to use quasi maximum likelihood estimation, which produces a more precise and significantly higher estimate of consumption insurance at 55%. For sub-groups delineated by age and education, differences between estimates are even starker. Monte Carlo experiments with non-Normal shocks demonstrate that our proposed likelihood-based approach is more accurate than GMM, especially given a small sample.

Keywords: consumption insurance; weighting schemes; quasi maximum likelihood
JEL codes: E21; C13; C33

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1 Introduction

How does idiosyncratic income risk impact consumption when households have limited access to insurance via formal markets or informal arrangements?¹ In a seminal paper, Blundell, Pistaferri, and Preston (2008) (BPP hereafter) construct a novel annual panel dataset of household income and (imputed) consumption for the Panel Study of Income Dynamics and employ generalized method of moments (GMM) to estimate the degree of consumption insurance based on their proposed model of the data. Numerous studies have followed their approach (e.g., Kaplan, Violante and Weidner 2014; Auclert, 2019). In this paper, we re-visit the estimation in BPP and propose an alternative likelihood-based approach.

Using diagonal weights for GMM, BPP find an estimate of consumption insurance with respect to permanent income shocks of 36%. We find that their estimate is affected by an error in their original code. In particular, not all sample moments are matched correctly to model-implied moments. For their dataset, this does not affect the empirical estimate very much (33% instead of 36%), but it does have a large effect on the performance of the estimator.² Of even greater concern, the BPP estimate of consumption insurance is quite imprecise and highly dependent on the weighting scheme used for GMM. Given optimal weights, we find a very different estimate of 67%. Furthermore, the lack of precision using diagonal weights and sensitivity to weighting scheme are even more striking for sub-group analysis based on age and education.

We propose instead to use quasi maximum likelihood estimation (QMLE), which produces a more precise and significantly higher estimate of consumption insurance compared to BPP at 55% for the same dataset. This approach avoids having to make a choice about weighting scheme and gains efficiency by implicitly imposing the model structure in fitting sample moments across time. By contrast, GMM using diagonal weights assumes independence of sample moments, while GMM using optimal weights allows for potential dependence, but does not take the full structure of the model into account in estimation and appears to be susceptible to serious biases and other problems in small samples.³ QMLE is made feasible in the panel setting by re-writing the BPP model as an unobserved components model and applying the Kalman filter to construct the likelihood under the assumption of Normality.⁴ Given the apparent non-Normality of the data

¹See Jappelli and Pistaferri (2011) for a comprehensive survey of this literature.

²For the Monte Carlo experiments considered in this paper, the error in the code leads to extremely inaccurate estimates for GMM using diagonal weights.

³See Altonji and Segal (1996) on the difficulties with GMM using optimal weights in a cross-sectional setting. Based on our Monte Carlo analysis, we find similar issues given a small sample in the BPP panel setting.

⁴Related to this approach, Primiceri and van Rens (2009) consider Bayesian estimation with flat and uninformative priors for an unobserved components model of household income and consumption in order

(see, for example, Guvenen, Karahan, Ozcan, and Song, 2015), we regard our estimation as quasi maximum likelihood (White, 1982) and demonstrate its performance relative to GMM via Monte Carlo experiments with non-Normal shocks.

For sub-group analysis, the differences in estimates for GMM and QMLE are even starker. Our proposed likelihood-based approach suggests significantly different estimates based on both age and education. As in BPP, college-educated households have more consumption insurance. However, our estimates are quite different than theirs and much more precise. For sub-groups based on age, BPP do not report estimates due to imprecision. Using QMLE, we find that younger households have significantly lower consumption insurance than older households.

Motivated by the empirical results, we conduct Monte Carlo experiments for two different sample sizes. Given the same sample size and missing data structure as for the full BPP dataset, we find that QMLE is the most accurate in terms of root mean squared error. GMM using diagonal weights is far less accurate, although GMM using optimal weights is closer in accuracy to QMLE. However, given a smaller sample size corresponding to one of the sub-groups, QMLE performs much better than GMM in both cases. The Monte Carlo results generally reconcile the differences in our sample estimates and suggest that QMLE estimates should be preferred to GMM, especially in small samples.

The rest of this paper is organized as follows: Section 2 describes the data and re-writes the BPP model as an unobserved components model for the purposes of likelihood-based estimation. Section 3 reports empirical results for the whole BPP dataset and sub-groups delineated by age and education. Section 4 presents our Monte Carlo analysis. Section 5 concludes.

2 Data, Model, and Estimation

In our empirical analysis, we use the dataset created by BPP. They consider the Panel Study of Income Dynamics (PSID) sample from 1978-1992 of continuously married couples headed by a male (with or without children) aged 30 to 65. The income variable is family disposable income, which includes transfers. They adopt a similar sample selection in the Consumer Expenditure Survey (CEX). Since CEX has detailed nondurable consumption data, while PSID primarily has only food expenditure consumption data, they impute nondurable consumption for each household per year by using the estimates of food demand from CEX. Their constructed dataset is then a panel of income and imputed nondurable consumption. To get idiosyncratic income and consumption, BPP regress income and consumption for households on demographic and ethnic factors, as well as

to examine changes in income inequality across households.

other income characteristics observable/known by consumers, and calculate residuals.⁵ Following BPP, it is these residual idiosyncratic income and consumption series that we model.

We re-write the original BPP model as an unobserved components model of household income and consumption with time-varying volatility for income and consumption shocks. In particular, idiosyncratic income and consumption for household i have the following form:

$$y_{i,t} = \tau_{i,t} + \epsilon_{i,t} + \theta\epsilon_{i,t-1}, \quad \epsilon_{i,t} \sim i.i.d.(0, \sigma_{\epsilon,t}^2), \quad (1)$$

$$c_{i,t} = \gamma_{\eta}\tau_{i,t} + \kappa_{i,t} + v_{i,t}, \quad v_{i,t} \sim i.i.d.(0, \sigma_{v,t}^2), \quad (2)$$

where $\tau_{i,t}$ is the common stochastic trend for income and consumption (“permanent income”), $\epsilon_{i,t}$ is the transitory income shock with moving-average parameter $|\theta| < 1$, $\kappa_{i,t}$ is an additional stochastic trend for consumption, and $v_{i,t}$ is the transitory consumption shock. The stochastic trends are specified as random walks:

$$\tau_{i,t} = \tau_{i,t-1} + \eta_{i,t}, \quad \eta_{i,t} \sim i.i.d.(0, \sigma_{\eta,t}^2), \quad (3)$$

$$\kappa_{i,t} = \kappa_{i,t-1} + \gamma_{\epsilon}\epsilon_{i,t} + u_{i,t}, \quad u_{i,t} \sim i.i.d.(0, \sigma_{u,t}^2). \quad (4)$$

while the structure of the time-varying volatility for each shock is assumed to be deterministic and the same as in BPP.⁶

In terms of economic interpretation, the transitory income shock, $\epsilon_{i,t}$, captures events such as a surprise bonus or temporary leave due illness, while the transitory consumption shock, $v_{i,t}$ could capture measurement error due to the imputation of nondurable consumption. The permanent income shock, $\eta_{i,t}$, reflects severe health shocks, promotion, or other idiosyncratic factors that result in a change in idiosyncratic permanent income, while the permanent shock to consumption, $u_{i,t}$, could reflect taste and preference shocks or other shocks to non-labor income, such as wealth shocks.

The key parameters that we focus on in our analysis are γ_{η} and γ_{ϵ} , which capture the impacts of permanent and transitory income shocks on permanent consumption. The implied “consumption insurance” against permanent income shocks is then $1 - \gamma_{\eta}$.

We propose estimating the model in (1)-(4) using a likelihood-based approach. In particular, we cast the model into state-space form (see the appendix) and assume that the

⁵In particular, in the BPP dataset, idiosyncratic income and consumption for households are calculated by removing the impact of observables such as education, race, family size, number of children, region, employment status, year and cohort effects, residence in large city, and presence of income recipients other than husband and wife.

⁶BPP generally allow different variances of shocks in each period, although they assume the variances of some shocks are the same across certain time periods.

shocks are normally distributed in order to use the Kalman filter to calculate the likelihood based on the prediction error decomposition. To the extent that the actual shocks are non-Normal, as suggested in some recent literature (see, for example, Guvenen, Karahan, Ozcan, and Song, 2015), our proposed approach can be thought of as quasi maximum likelihood estimation (QMLE) following White (1982). We will consider how well QMLE works relative to GMM in this setting with our Monte Carlo analysis in Section 4.

For purposes of comparison, we also consider GMM estimation using the same moment conditions as BPP. We employ two approaches for weighting the moment conditions, the diagonally weighted minimum distance (DWMD) approach taken in BPP and the optimal minimum distance (OMD) approach. DWMD generalizes an equally weighted minimum distance approach, but allows for heteroskedasticity, while OMD allows for covariance between moment conditions in the weighting matrix.

3 Empirical Results

Table 1 reports estimates for the parameters related to the responses of consumption to income shocks based on the whole sample of households.⁷ The QMLE estimates are the most precise and lie in between the GMM estimates for both parameters. The DWMD estimates are close to those reported in BPP Table 6. However, there are differences due to an error in the original BPP code.⁸ The OMD estimates are very different from the DWMD estimates, suggesting a high sensitivity to weighting of moments. The QMLE estimate for the effect of permanent income shocks is closer to the OMD estimate, with both implying a significantly higher degree of consumption insurance than reported in BPP – between 55-67% rather than 36%. The estimates for the effect of a transitory income shock are more similar across estimation methods, however they are only significantly positive for QMLE and OMD given greater precision compared to DWMD.

Table 2 reports estimates of the same parameters of interest for sub-groups of households delineated by education and age. The results again vary substantially by estimation method, with QMLE almost always being the most precise and generally in between the GMM estimates. The DWMD estimates for education are again similar to those in BPP Table 6, with differences due to the original code error noted above. Meanwhile, for age, the DWMD estimates are highly imprecise.⁹ In terms of the effect of permanent income shocks, only the QMLE results are consistent with higher consumption insur-

⁷The full set of estimates for all of the model parameters are presented in the appendix.

⁸In the BPP code, *MD_AER.prg*, available from the AER website, there is a misplaced transpose on line 289. If we use their original code, we obtain identical estimates based on DWMD to those reported in BPP Table 6. However, the misplaced transpose leads to a mismatching of some model-implied moments to sample moments and produces substantially different estimates on average in Monte Carlo analysis.

⁹BPP do not report estimates delineated by age, but note their imprecision in footnote 31 of their paper.

TABLE 1. ESTIMATES FOR CONSUMPTION RESPONSES

| Parameter | QMLE | GMM | |
|-------------------|--------------------------------|---------------------------------|--------------------------------|
| | | DWMD | OMD |
| γ_η | 0.45 (0.02) [0.41, 0.50] | 0.67 (0.09) [0.49, 0.85] | 0.33 (0.03) [0.27, 0.39] |
| γ_ϵ | 0.04 (0.01) [0.02, 0.06] | 0.03 (0.04) [-0.05, 0.11] | 0.07 (0.03) [0.01, 0.13] |

Notes: The table reports point estimates, with standard deviations in parentheses and 95% confidence intervals in square brackets for the whole sample of households. QMLE confidence intervals are based on inverted likelihood ratio tests. GMM confidence intervals are based on inverted t tests.

ance for both college-educated and older households. GMM based on OMD suggests a counter-intuitive result that households with no college education have substantially higher consumption insurance of 75%. GMM based on DWMD implies very low consumption insurance for both young and old households compared to the estimate for the whole sample, with young households having higher consumption insurance than older households at 27% versus 15%. Again, the estimates display a high sensitivity to weighting of moments, particularly with implied consumption insurance for households with no college education ranging from 5% for DWMD to 75% for OMD and for older households ranging from 15% for DWMD to 81% for OMD. The sub-group estimates for QMLE imply relatively high, but moderate levels of consumption insurance of 44% versus 64% based on education and 48% versus 70% based on age. The QMLE estimates for the effect of a transitory income shock are always more precise than the GMM estimates, which are sometimes counter-intuitively negative.

4 Monte Carlo Analysis

Although the QMLE estimates in Tables 1 and 2 are the most precise, it is an open question whether this is, in some sense, a false precision, perhaps due to the Normality assumption made in constructing the likelihood for the model. To address this question, we consider a Monte Carlo experiment where the data generating process corresponds to the BPP model with shocks drawn from their empirical distribution based on the estimated model in the previous section.¹⁰ Notably, the empirically-distributed shocks display an extremely high degree of kurtosis compared to Normal shocks. We draw, with replacement, from the

¹⁰Given QMLE parameter estimates, we employ the Kalman filter to extract inferences about the underlying permanent and transitory income and consumption shocks.

TABLE 2. ESTIMATES FOR SUB-GROUP RESPONSES

| Parameter | QMLE | GMM | |
|-------------------|---------------------------------|----------------------------------|----------------------------------|
| | | DWMD | OMD |
| No COLLEGE | | | |
| γ_η | 0.56 (0.02) [0.52, 0.60] | 0.95 (0.17) [0.61, 1.29] | 0.25 (0.04) [0.17,0.33] |
| γ_ϵ | 0.05 (0.02) [0.01, 0.09] | 0.07 (0.06) [-0.05, 0.19] | 0.18 (0.03) [0.12,0.24] |
| COLLEGE | | | |
| γ_η | 0.36 (0.03) [0.30, 0.42] | 0.47 (0.09) [0.29, 0.65] | 0.47 (0.04) [0.39, 0.55] |
| γ_ϵ | 0.04 (0.02) [0.00, 0.08] | -0.01 (0.05) [-0.11, 0.09] | -0.12 (0.03) [-0.18,-0.06] |
| YOUNGER (30-47) | | | |
| γ_η | 0.52 (0.03) [0.46, 0.58] | 0.73 (0.11) [0.51, 0.95] | 0.52 (0.05) [0.42, 0.62] |
| γ_ϵ | 0.01 (0.02) [-0.03, 0.05] | -0.02 (0.07) [-0.05, 0.19] | 0.02 (0.04) [-0.06,0.10] |
| OLDER (48-65) | | | |
| γ_η | 0.30 (0.04) [0.22, 0.38] | 0.85 (0.22) [0.44, 1.28] | 0.19 (0.03) [0.13, 0.25] |
| γ_ϵ | 0.08 (0.02) [0.04, 0.12] | 0.06 (0.05) [-0.04, 0.16] | 0.15 (0.03) [0.09,0.21] |

Notes: The table reports point estimates, with standard deviations in parentheses and 95% confidence intervals in square brackets for the four sub-groups. QMLE confidence intervals are based on inverted likelihood ratio tests. GMM confidence intervals are based on inverted t tests. For the sub-group based on education, there are 883 households classified as 'No College' and 882 households as 'College'. For age, there are 1,413 households classified as 'Younger' and 708 households classified as 'Older'.

TABLE 3. PROPERTIES OF ESTIMATORS GIVEN LARGE SAMPLE

| Parameter | Estimator Property | QMLE | GMM | |
|--------------------------|----------------------|-------|-------|------|
| | | | DWMD | OMD |
| $\gamma_\eta = 0.50$ | RMSE | 0.06 | 0.12 | 0.07 |
| | Bias | 0.05 | 0.05 | 0.03 |
| | Standard Deviation | 0.03 | 0.11 | 0.06 |
| | Difference from QMLE | - | 0.11 | 0.06 |
| $\gamma_\epsilon = 0.10$ | RMSE | 0.03 | 0.03 | 0.03 |
| | Bias | -0.03 | -0.01 | 0.00 |
| | Standard Deviation | 0.00 | 0.03 | 0.03 |
| | Difference from QMLE | - | 0.04 | 0.04 |

Notes: RMSE, bias, standard deviation, and (root mean squared) difference from QMLE for different estimators are based on averages across 2,500 simulations with sample size $T \times N = 15 \times 1765$.

empirical shocks for each time period and use the BPP model with stylized parameters ($\gamma_\eta = 0.50$, $\gamma_\epsilon = 0.10$, $\theta = 0.20$) to construct artificial panels of idiosyncratic income and consumption data with the same sample size (N and T) and structure in terms of missing observations as in the BPP dataset.

To evaluate the accuracy of a particular estimator, we consider root mean squared error (RMSE).¹¹ We also report on the underlying sources of the overall RMSE in terms of bias and standard deviation of an estimator, as well as the root mean squared differences of the GMM estimators compared to QMLE. Each statistic is calculated based on averaging across 2,500 simulations. We focus on results for our key parameters of interest, γ_η and γ_ϵ .

Table 3 reports on the accuracy of different estimators given a large sample with the same structure as the whole BPP dataset. QMLE performs best in terms of RMSE, although OMD is quite similar. DWMD is much less accurate in terms of γ_η . The main reason for the strong performance of QMLE is a much lower standard deviation of the estimator for both parameters, although this is somewhat offset by more bias than OMD in particular. Notably, the GMM estimators also differ considerably from QMLE in a given sample, with root mean squared differences of a similar magnitude to their RMSEs.

Looking back to Table 1, the Monte Carlo results in Table 3 help explain some of the key differences across sample estimates. In particular, the relative precision of the esti-

¹¹Nakata and Tonetti (2015) conduct Monte Carlo analysis to evaluate the RMSE of likelihood-based Bayesian estimators of income risk in a univariate setting and find that they perform well.

mates is consistent with the standard deviations of the estimators, with QMLE being most precise and DWMD being least precise. The difference between the QMLE and OMD estimates for γ_η can be partly reconciled by the somewhat higher bias for QMLE and the substantial root mean squared difference from QMLE for OMD. However, the key point is that the QMLE estimator is the most accurate overall based on RMSE and, assuming any bias is similar to the Monte Carlo result, implies consumption insurance between 55-64%.¹² Meanwhile, the QMLE and OMD estimates for γ_ϵ can be reconciled entirely by a difference in bias similar to the Monte Carlo results, with a small, but non-zero effect of transitory income shocks on consumption.

Table 4 reports on the accuracy of different estimators given a smaller sample with the same structure in terms of missing observations as the sample of older households in the BPP dataset. Given well-known concerns about OMD in small samples for cross-sectional analysis (see Altonji and Segal, 1996), our aim is to understand how the performance of QMLE and GMM compare given a smaller sample in a panel setting, with the effective sample size being smallest for older households.¹³ As in the large sample case, QMLE performs best in terms of RMSE, but the improvements over GMM are much more dramatic, especially compared to OMD. The lower RMSE again results from a comparatively low standard deviation of the QMLE estimator for both parameters, with similar biases as before in almost every case. The only change in bias is a large increase for γ_η with DWMD, leading it to being the worst performing estimator again for this key parameter of interest. But the striking difference from Table 3 is the severe deterioration of OMD, which even performs worse in terms of RMSE and standard deviation than DWMD for γ_ϵ . By contrast, the accuracy of QMLE is almost as good as in the large sample case. Notably, it is also almost three times more accurate than OMD for γ_η and twice as accurate for γ_ϵ . Meanwhile, the GMM estimators differ from QMLE, with root mean squared differences again of a similar magnitude to their RMSEs.

Looking back to Table 2, the results in Table 4 help explain why the sample estimates are so different across estimation methods in most cases and raise strong doubts about the accuracy of the GMM estimates in the smaller sample setting. Furthermore, unlike with OMD, the Monte Carlo results provide confidence in the precision of the QMLE estimates. The main takeaway is that consumption insurance appears to be significantly higher for college-educated and older households, with about 20 percentage point higher insurance than the comparison sub-groups in both cases. Again, assuming any bias is similar to the Monte Carlo result, implied consumption insurance is between 45-53% for no college versus 63-75% for college and 47-59% for young versus 67-83% for older households.

¹²It is possible to consider a formal bootstrap correction for bias, but we leave this for future research.

¹³In particular, N equals 708 for older households, 1,413 for younger households, 883 for households with no college education, 882 for college-educated households, and 1,765 for the whole sample.

TABLE 4. PROPERTIES OF ESTIMATORS GIVEN SMALL SAMPLE

| Parameter | Estimator Property | QMLE | GMM | |
|--------------------------|-------------------------|-------|-------|------|
| | | | DWMD | OMD |
| $\gamma_\eta = 0.50$ | RMSE | 0.07 | 0.26 | 0.20 |
| | Bias | 0.05 | 0.12 | 0.03 |
| | SD of γ_η | 0.05 | 0.23 | 0.20 |
| | RMSD (relative to MLE) | - | 0.24 | 0.20 |
| $\gamma_\epsilon = 0.10$ | RMSE | 0.04 | 0.06 | 0.08 |
| | Bias | -0.03 | -0.01 | 0.00 |
| | SD of γ_ϵ | 0.03 | 0.06 | 0.08 |
| | RMSD (relative to MLE) | - | 0.06 | 0.08 |

Notes: The table reports RMSE, relative RMSE and bias for different estimators across 2,500 simulations, where the sample size is $T \times N = 15 \times 708$ and shocks are drawn from empirical distributions, with replacement. However note 42 percent of the income data and 50 percent of the consumption data is missing in the sample.

For the effects of transitory income shocks, the QMLE estimates for γ_ϵ are, as with the whole sample, consistent with a small, but non-zero effect for all sub-groups, at least when taking the apparent downward bias into account.

5 Conclusion

In this paper, we have examined the robustness of Blundell, Pistaferri, and Preston's (2008) finding of low consumption insurance with respect to permanent income shocks. We find that their result is not robust to different estimation methods, with quasi maximum likelihood implying a much higher and more precise estimate of consumption insurance. Estimates for sub-groups are also sensitive to estimation method, with quasi maximum likelihood estimates again being the most precise and suggesting intuitive heterogeneity across households grouped by age and education. Monte Carlo analysis allowing for non-Normal shocks supports the greater accuracy of quasi maximum likelihood estimation versus the GMM approach taken in Blundell, Pistaferri, and Preston (2008).

We believe our paper makes two significant contributions to the literature on household consumption insurance. First, we provide evidence that consumption insurance is higher than previously reported for Blundell, Pistaferri, and Preston's (2008) dataset, at least based on their model.¹⁴ Second, we show the feasibility of quasi maximum likeli-

¹⁴Kaplan and Violante (2010) argue that Blundell, Pistaferri, and Preston's (2008) estimate for consump-

hood estimation of an unobserved components model in a panel setting and recommend using it in practice.

tion insurance is downward biased due to model misspecification. Thus, our finding of higher consumption insurance for the dataset is consistent with the implication of Kaplan and Violante (2010), but for the different reason of imprecision of the GMM estimator using diagonal weights rather than model misspecification.

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A State-Space Representation

In this appendix, we present the state-space representation of the BPP model described in Section 2.

Suppressing the individual-specific subscript for simplicity, the observation equation is

$$\tilde{\mathbf{y}}_t = \mathbf{H} \boldsymbol{\beta}_t,$$

where

$$\tilde{\mathbf{y}}_t = \begin{bmatrix} y_t \\ c_t \end{bmatrix}, \mathbf{H} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & \gamma_\eta & 1 \end{bmatrix}, \text{ and } \boldsymbol{\beta}_t = \begin{bmatrix} y_t - \tau_t \\ c_t - \gamma_\eta \tau_t - \kappa_t \\ \tau_t \\ \kappa_t \end{bmatrix}.$$

The state equation is

$$\boldsymbol{\beta}_t = \mathbf{F} \boldsymbol{\beta}_{t-1} + \tilde{\mathbf{v}}_t,$$

where

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tilde{\mathbf{v}}_t = \begin{bmatrix} \epsilon_t + \theta \epsilon_{t-1} \\ v_t \\ \eta_t \\ u_t + \gamma_\epsilon \epsilon_t \end{bmatrix},$$

and the covariance matrix of $\tilde{\mathbf{v}}_t$, \mathbf{Q} , is given by

$$\mathbf{Q} = \begin{pmatrix} \sigma_\epsilon^2(1 + \theta^2) & 0 & 0 & \gamma_\epsilon \sigma_\epsilon^2 \\ 0 & \sigma_v^2 & 0 & 0 \\ 0 & 0 & \sigma_\eta^2 & 0 \\ \gamma_\epsilon \sigma_\epsilon^2 & 0 & 0 & \sigma_u^2 + \gamma_\epsilon^2 \sigma_\epsilon^2 \end{pmatrix}.$$

Given the state-space representation and an assumption of Normality, we can use the Kalman filter to calculate the likelihood for the BPP model based on the prediction error decomposition. In addition, the Kalman filter can be easily adapted to handle missing observations, which are prevalent in the BPP dataset. Given the likelihood, we use numerical optimization to conduct QMLE. Meanwhile, for GMM estimation, we follow the approach in BPP. See their paper and appendix for details.

B Full Sets of Estimates

In this appendix, we report the full set of point estimates, with standard deviations in parentheses, for different estimation methods considered in the main text.

TABLE A1. ESTIMATES FOR WHOLE SAMPLE

| Parameter | | QMLE | GMM | |
|-------------------|---------|-------------|-------------|-------------|
| | | | DWMD | OMD |
| σ_η | 1979-81 | 0.14 (0.00) | 0.10 (0.00) | 0.12 (0.00) |
| . | 1982 | 0.10 (0.01) | 0.14 (0.00) | 0.14 (0.00) |
| . | 1983 | 0.13 (0.01) | 0.17 (0.01) | 0.13 (0.00) |
| . | 1984 | 0.12 (0.01) | 0.17 (0.01) | 0.08 (0.00) |
| . | 1985 | 0.13 (0.01) | 0.17 (0.01) | 0.14 (0.01) |
| . | 1986 | 0.10 (0.01) | 0.15 (0.01) | 0.12 (0.00) |
| . | 1987 | 0.13 (0.01) | 0.17(0.01) | 0.12 (0.00) |
| . | 1988 | 0.06 (0.02) | 0.13 (0.01) | 0.14 (0.01) |
| . | 1989 | 0.12 (0.01) | 0.13 (0.01) | 0.12 (0.00) |
| . | 1990-92 | 0.12 (0.01) | 0.12 (0.00) | 0.10 (0.00) |
| σ_ϵ | 1979 | 0.18 (0.01) | 0.19 (0.01) | 0.14 (0.00) |
| . | 1980 | 0.19 (0.01) | 0.17 (0.00) | 0.12 (0.00) |
| . | 1981 | 0.19 (0.01) | 0.17 (0.00) | 0.14 (0.00) |
| . | 1982 | 0.20 (0.01) | 0.17 (0.00) | 0.16 (0.00) |
| . | 1983 | 0.19 (0.01) | 0.16 (0.00) | 0.14 (0.00) |
| . | 1984 | 0.20 (0.01) | 0.19 (0.00) | 0.16 (0.00) |
| . | 1985 | 0.25 (0.01) | 0.21 (0.01) | 0.17 (0.00) |
| . | 1986 | 0.24 (0.01) | 0.21 (0.01) | 0.18 (0.00) |
| . | 1987 | 0.24 (0.01) | 0.21 (0.01) | 0.18 (0.00) |
| . | 1988 | 0.22 (0.01) | 0.20 (0.01) | 0.16 (0.00) |
| . | 1989 | 0.21 (0.01) | 0.20 (0.01) | 0.17 (0.00) |
| . | 1990-92 | 0.23 (0.00) | 0.21 (0.00) | 0.16 (0.00) |
| σ_u | | 0.08 (0.00) | 0.10 (0.00) | 0.09 (0.00) |
| σ_v | 1979 | 0.26 (0.01) | 0.25 (0.01) | 0.23 (0.00) |
| . | 1980 | 0.24 (0.01) | 0.23 (0.01) | 0.20 (0.00) |
| . | 1981 | 0.24 (0.01) | 0.23 (0.01) | 0.20 (0.00) |
| . | 1982 | 0.28 (0.01) | 0.25 (0.01) | 0.21 (0.01) |
| . | 1983 | 0.26 (0.01) | 0.26 (0.01) | 0.20 (0.00) |
| . | 1984 | 0.34 (0.01) | 0.31 (0.02) | 0.25 (0.01) |
| . | 1985 | 0.30 (0.01) | 0.28 (0.02) | 0.25 (0.01) |
| . | 1986 | 0.27 (0.01) | 0.26 (0.01) | 0.23 (0.01) |
| . | 1989 | 0.31 (0.01) | NA* | NA* |
| . | 1990-92 | 0.28 (0.00) | 0.26 (0.01) | 0.21 (0.00) |
| θ | | 0.19 (0.01) | 0.11 (0.03) | 0.12 (0.02) |
| γ_ϵ | | 0.04 (0.01) | 0.03 (0.04) | 0.07 (0.03) |
| γ_η | | 0.45 (0.02) | 0.67 (0.09) | 0.33 (0.03) |

*GMM estimation is based on growth rates and, therefore, cannot estimate a variance for transitory consumption in 1989 given missing consumption data in 1988.

TABLE A2. ESTIMATES FOR NO COLLEGE SUB-GROUP

| Parameter | | QMLE | GMM | |
|-------------------|---------|-------------|-------------|-------------|
| | | | DWMD | OMD |
| σ_η | 1979-81 | 0.13 (0.01) | 0.08 (0.00) | 0.12 (0.00) |
| . | 1982 | 0.08 (0.02) | 0.12 (0.01) | 0.11 (0.00) |
| . | 1983 | 0.13 (0.01) | 0.18 (0.01) | 0.12 (0.00) |
| . | 1984 | 0.13 (0.01) | 0.18 (0.01) | 0.10 (0.00) |
| . | 1985 | 0.11 (0.02) | 0.17 (0.01) | 0.08 (0.00) |
| . | 1986 | 0.10 (0.02) | 0.13 (0.01) | 0.14 (0.00) |
| . | 1987 | 0.11 (0.02) | 0.14 (0.01) | 0.12 (0.00) |
| . | 1988 | 0.05 (0.04) | 0.11 (0.01) | 0.09 (0.00) |
| . | 1989 | 0.10 (0.02) | 0.10 (0.01) | 0.10 (0.00) |
| . | 1990-92 | 0.11 (0.01) | 0.10 (0.01) | 0.06 (0.00) |
| σ_ϵ | 1979 | 0.21 (0.01) | 0.21(0.01) | 0.13 (0.00) |
| . | 1980 | 0.20 (0.01) | 0.18 (0.01) | 0.15 (0.00) |
| . | 1981 | 0.20 (0.01) | 0.19 (0.01) | 0.15 (0.00) |
| . | 1982 | 0.22 (0.01) | 0.20 (0.01) | 0.15 (0.00) |
| . | 1983 | 0.22 (0.01) | 0.19 (0.01) | 0.17 (0.00) |
| . | 1984 | 0.22 (0.01) | 0.20 (0.01) | 0.18 (0.00) |
| . | 1985 | 0.22 (0.01) | 0.19 (0.01) | 0.19 (0.00) |
| . | 1986 | 0.23 (0.01) | 0.22 (0.01) | 0.17 (0.00) |
| . | 1987 | 0.25 (0.01) | 0.23 (0.01) | 0.19 (0.00) |
| . | 1988 | 0.23 (0.01) | 0.22 (0.01) | 0.15 (0.00) |
| . | 1989 | 0.24 (0.01) | 0.23 (0.01) | 0.19 (0.00) |
| . | 1990-92 | 0.24 (0.01) | 0.23 (0.01) | 0.18 (0.00) |
| σ_u | | 0.08 (0.00) | 0.08 (0.01) | 0.09 (0.00) |
| σ_v | 1979 | 0.30 (0.01) | 0.28 (0.01) | 0.20 (0.01) |
| . | 1980 | 0.27 (0.01) | 0.27 (0.01) | 0.24 (0.01) |
| . | 1981 | 0.25 (0.01) | 0.25 (0.01) | 0.20 (0.00) |
| . | 1982 | 0.32 (0.01) | 0.28 (0.01) | 0.22 (0.00) |
| . | 1983 | 0.29 (0.01) | 0.28 (0.02) | 0.21 (0.00) |
| . | 1984 | 0.39 (0.01) | 0.35 (0.03) | 0.22 (0.01) |
| . | 1985 | 0.35 (0.01) | 0.33 (0.03) | 0.23 (0.01) |
| . | 1986 | 0.31 (0.01) | 0.29 (0.01) | 0.24 (0.01) |
| . | 1989 | 0.35 (0.01) | NA* | NA* |
| . | 1990-92 | 0.32 (0.01) | 0.29 (0.01) | 0.22 (0.00) |
| θ | | 0.19 (0.02) | 0.13 (0.03) | 0.19 (0.02) |
| γ_ϵ | | 0.05 (0.02) | 0.07 (0.06) | 0.18 (0.03) |
| γ_η | | 0.56 (0.02) | 0.95 (0.17) | 0.25 (0.04) |

*GMM estimation is based on growth rates and, therefore, cannot estimate a variance for transitory consumption in 1989 given missing consumption data in 1988.

TABLE A3. ESTIMATES FOR COLLEGE SUB-GROUP

| Parameter | | QMLE | GMM | |
|-------------------|---------|-------------|--------------|--------------|
| | | | DWMD | OMD |
| σ_η | 1979-81 | 0.15 (0.01) | 0.10 (0.01) | 0.09 (0.00) |
| . | 1982 | 0.12 (0.01) | 0.16 (0.01) | 0.16 (0.00) |
| . | 1983 | 0.11 (0.01) | 0.15 (0.01) | 0.11 (0.00) |
| . | 1984 | 0.10 (0.01) | 0.13 (0.01) | 0.06 (0.00) |
| . | 1985 | 0.15 (0.02) | 0.15 (0.01) | NA* |
| . | 1986 | 0.09 (0.02) | 0.18 (0.01) | 0.14 (0.00) |
| . | 1987 | 0.15 (0.01) | 0.19 (0.01) | 0.12 (0.00) |
| . | 1988 | 0.07 (0.02) | 0.14 (0.01) | 0.18 (0.00) |
| . | 1989 | 0.13 (0.01) | 0.17 (0.01) | 0.11 (0.00) |
| . | 1990-92 | 0.13 (0.01) | 0.15 (0.01) | 0.09 (0.00) |
| σ_ϵ | 1979 | 0.15 (0.01) | 0.17 (0.01) | 0.13 (0.00) |
| . | 1980 | 0.17 (0.01) | 0.17 (0.01) | 0.12 (0.00) |
| . | 1981 | 0.17 (0.01) | 0.16 (0.00) | 0.09 (0.00) |
| . | 1982 | 0.18 (0.01) | 0.15 (0.00) | 0.10 (0.00) |
| . | 1983 | 0.17 (0.01) | 0.14 (0.00) | 0.12 (0.00) |
| . | 1984 | 0.18 (0.01) | 0.17 (0.01) | 0.15 (0.00) |
| . | 1985 | 0.27 (0.01) | 0.22 (0.01) | 0.13 (0.00) |
| . | 1986 | 0.24 (0.01) | 0.21 (0.01) | 0.11 (0.00) |
| . | 1987 | 0.23 (0.01) | 0.21 (0.01) | 0.12 (0.00) |
| . | 1988 | 0.21 (0.01) | 0.19 (0.01) | 0.15 (0.00) |
| . | 1989 | 0.18 (0.01) | 0.15 (0.01) | 0.15 (0.00) |
| . | 1990-92 | 0.21 (0.01) | 0.19 (0.00) | 0.15 (0.00) |
| σ_u | | 0.08 (0.00) | 0.11 (0.00) | 0.09 (0.00) |
| σ_v | 1979 | 0.22 (0.01) | 0.21 (0.01) | 0.19 (0.00) |
| . | 1980 | 0.20 (0.01) | 0.18 (0.01) | 0.16 (0.00) |
| . | 1981 | 0.22 (0.01) | 0.22 (0.01) | 0.17 (0.00) |
| . | 1982 | 0.23 (0.01) | 0.21 (0.01) | 0.19 (0.00) |
| . | 1983 | 0.24 (0.01) | 0.24 (0.01) | 0.18 (0.00) |
| . | 1984 | 0.29 (0.01) | 0.27 (0.02) | 0.23 (0.01) |
| . | 1985 | 0.25 (0.01) | 0.23 (0.01) | 0.22 (0.01) |
| . | 1986 | 0.24 (0.01) | 0.22 (0.01) | 0.19 (0.00) |
| . | 1989 | 0.27 (0.01) | NA** | NA** |
| . | 1990-92 | 0.24 (0.01) | 0.24 (0.01) | 0.20 (0.00) |
| θ | | 0.19 (0.02) | 0.11 (0.03) | 0.13 (0.03) |
| γ_ϵ | | 0.04 (0.02) | -0.01 (0.05) | -0.12 (0.03) |
| γ_η | | 0.36 (0.03) | 0.47 (0.09) | 0.47 (0.04) |

*OMD estimates a negative variance of the permanent income shock in 1985.

**GMM estimation is based on growth rates and, therefore, cannot estimate a variance for transitory consumption in 1989 given missing consumption data in 1988.

TABLE A4. ESTIMATES FOR YOUNGER SUB-GROUP

| Parameter | | QMLE | GMM | |
|-------------------|---------|-------------|--------------|-------------|
| | | | DWMD | OMD |
| σ_η | 1979-81 | 0.13 (0.00) | 0.10 (0.00) | 0.09 (0.00) |
| . | 1982 | 0.10 (0.01) | 0.15 (0.00) | 0.13 (0.00) |
| . | 1983 | 0.12 (0.01) | 0.16 (0.01) | 0.14 (0.00) |
| . | 1984 | 0.11 (0.01) | 0.16 (0.01) | 0.10 (0.00) |
| . | 1985 | 0.07 (0.02) | 0.13 (0.01) | 0.08 (0.00) |
| . | 1986 | 0.11 (0.01) | 0.17 (0.01) | 0.13 (0.00) |
| . | 1987 | 0.13 (0.01) | 0.17 (0.01) | 0.12 (0.00) |
| . | 1988 | 0.08 (0.02) | 0.13 (0.01) | 0.14 (0.01) |
| . | 1989 | 0.10 (0.01) | 0.14 (0.01) | 0.09 (0.00) |
| . | 1990-92 | 0.11 (0.01) | 0.13 (0.00) | 0.09 (0.00) |
| σ_ϵ | 1979 | 0.17 (0.01) | 0.18 (0.00) | 0.15 (0.00) |
| . | 1980 | 0.17 (0.01) | 0.16 (0.00) | 0.13 (0.00) |
| . | 1981 | 0.18 (0.01) | 0.16 (0.00) | 0.12 (0.00) |
| . | 1982 | 0.19 (0.01) | 0.16 (0.00) | 0.14 (0.00) |
| . | 1983 | 0.18 (0.01) | 0.16 (0.00) | 0.14 (0.00) |
| . | 1984 | 0.18 (0.01) | 0.15 (0.00) | 0.13 (0.00) |
| . | 1985 | 0.20 (0.01) | 0.18 (0.01) | 0.15 (0.00) |
| . | 1986 | 0.21 (0.01) | 0.17 (0.01) | 0.15 (0.00) |
| . | 1987 | 0.23 (0.01) | 0.21 (0.01) | 0.17 (0.00) |
| . | 1988 | 0.20 (0.01) | 0.18 (0.00) | 0.15 (0.00) |
| . | 1989 | 0.19 (0.01) | 0.16 (0.00) | 0.14 (0.00) |
| . | 1990-92 | 0.20 (0.00) | 0.18 (0.00) | 0.14 (0.00) |
| σ_u | | 0.07 (0.00) | 0.10 (0.01) | 0.09 (0.00) |
| σ_v | 1979 | 0.28 (0.01) | 0.27 (0.01) | 0.20 (0.01) |
| . | 1980 | 0.25 (0.01) | 0.23 (0.01) | 0.18 (0.00) |
| . | 1981 | 0.24 (0.01) | 0.24 (0.01) | 0.19 (0.01) |
| . | 1982 | 0.28 (0.01) | 0.25 (0.01) | 0.21 (0.01) |
| . | 1983 | 0.26 (0.01) | 0.25 (0.01) | 0.20 (0.00) |
| . | 1984 | 0.36 (0.01) | 0.33 (0.02) | 0.23 (0.01) |
| . | 1985 | 0.33 (0.01) | 0.32 (0.03) | 0.26 (0.01) |
| . | 1986 | 0.27 (0.01) | 0.24 (0.01) | 0.21 (0.00) |
| . | 1989 | 0.34 (0.01) | NA* | NA* |
| . | 1990-92 | 0.27 (0.01) | 0.25 (0.01) | 0.21 (0.00) |
| θ | | 0.19 (0.02) | 0.11 (0.04) | 0.17 (0.02) |
| γ_ϵ | | 0.01 (0.01) | -0.02 (0.07) | 0.02 (0.04) |
| γ_η | | 0.52 (0.03) | 0.73 (0.11) | 0.52 (0.05) |

*GMM estimation is based on growth rates and, therefore, cannot estimate a variance for transitory consumption in 1989 given missing consumption data in 1988.

TABLE A5. ESTIMATES FOR OLDER SUB-GROUP

| Parameter | | QMLE | GMM | |
|-------------------|---------|-------------|-------------|-------------|
| | | | DWMD | OMD |
| σ_η | 1979-81 | 0.14 (0.01) | 0.08 (0.00) | 0.14 (0.00) |
| . | 1982 | 0.07 (0.03) | 0.12 (0.01) | 0.10 (0.00) |
| . | 1983 | 0.13 (0.02) | 0.18 (0.01) | 0.12 (0.00) |
| . | 1984 | 0.22 (0.02) | 0.17 (0.01) | 0.12 (0.00) |
| . | 1985 | 0.05 (0.05) | 0.16 (0.01) | 0.16 (0.01) |
| . | 1986 | 0.12 (0.02) | 0.09 (0.01) | NA* |
| . | 1987 | 0.00 (0.00) | 0.17 (0.01) | 0.12 (0.00) |
| . | 1988 | 0.00** | 0.11 (0.01) | 0.04 (0.00) |
| . | 1989 | 0.11 (0.02) | 0.13 (0.01) | 0.16 (0.01) |
| . | 1990-92 | 0.15 (0.01) | 0.08 (0.01) | 0.15 (0.00) |
| σ_ϵ | 1979 | 0.22 (0.01) | 0.22 (0.01) | 0.09 (0.00) |
| . | 1980 | 0.23 (0.01) | 0.20 (0.01) | 0.10 (0.00) |
| . | 1981 | 0.20 (0.01) | 0.19 (0.01) | 0.15 (0.00) |
| . | 1982 | 0.22 (0.01) | 0.19 (0.01) | 0.15 (0.00) |
| . | 1983 | 0.23 (0.01) | 0.17 (0.01) | 0.11 (0.00) |
| . | 1984 | 0.25 (0.01) | 0.23 (0.01) | 0.18 (0.00) |
| . | 1985 | 0.31 (0.01) | 0.27 (0.02) | 0.18 (0.00) |
| . | 1986 | 0.28 (0.01) | 0.27 (0.01) | 0.21 (0.00) |
| . | 1987 | 0.26 (0.01) | 0.22 (0.01) | 0.18 (0.00) |
| . | 1988 | 0.25 (0.01) | 0.23 (0.01) | 0.18 (0.00) |
| . | 1989 | 0.26 (0.01) | 0.25 (0.02) | 0.18 (0.00) |
| . | 1990-92 | 0.26 (0.00) | 0.26 (0.01) | 0.19 (0.00) |
| σ_u | | 0.08 (0.00) | 0.06 (0.01) | 0.10 (0.00) |
| σ_v | 1979 | 0.24 (0.01) | 0.22 (0.01) | 0.16 (0.00) |
| . | 1980 | 0.22 (0.01) | 0.23 (0.01) | 0.16 (0.00) |
| . | 1981 | 0.23 (0.01) | 0.22 (0.01) | 0.18 (0.00) |
| . | 1982 | 0.27 (0.01) | 0.25 (0.01) | 0.13 (0.00) |
| . | 1983 | 0.27 (0.01) | 0.27 (0.02) | 0.21 (0.00) |
| . | 1984 | 0.31 (0.01) | 0.27 (0.01) | 0.20 (0.01) |
| . | 1985 | 0.24 (0.01) | 0.23 (0.01) | 0.21 (0.00) |
| . | 1986 | 0.26 (0.01) | 0.25 (0.01) | 0.22 (0.00) |
| . | 1989 | 0.27 (0.01) | NA*** | NA*** |
| . | 1990-92 | 0.29 (0.01) | 0.29 (0.01) | 0.20 (0.01) |
| θ | | 0.20 (0.02) | 0.11 (0.03) | 0.17 (0.02) |
| γ_ϵ | | 0.08 (0.02) | 0.06 (0.05) | 0.15 (0.03) |
| γ_η | | 0.30 (0.04) | 0.85 (0.22) | 0.19 (0.03) |

*OMD estimates a negative variance of the permanent income shock in 1986.

**QMLE estimates a standard deviation of the permanent income shock of zero in 1988. We re-estimate the model with this restriction.

***GMM estimation is based on growth rates and, therefore, cannot estimate a variance for transitory consumption in 1989 given missing consumption data in 1988.